Economic Pipe Diameter

Typically, a pipe system involves six variables:

1) Pressure drop, ΔP
2) Flow rate, Q
3) Pipe diameter, D
4) Pipe length, L
5) Surface roughness, ε
6) Fluid viscosity, μ

Generally, in a typical pipe system problem, one of the first three variables (ΔP, Q or D) are calculated. Other three variables are determined based on the system requirements or assumed to start the analysis of the system.

Cost is another selection parameter of the pipe system. The pipe diameter is an important variable affecting the cost of the system. The larger the pipe diameter, the greater the initial cost. On the other hand, fluid flowing through a small diameter pipe undergoes a larger friction loss and hence a larger pump is required. A larger pump means greater initial cost but less operating cost. Hence, the pipe diameter should be determined such that the total cost (initial + operating cost) of the system is minimum. This diameter is called the optimum economic diameter, D_{opt}. 
**Optimum Economic Diameter, \( D_{\text{opt}} \)**

The optimum economic pipe diameter is the diameter that minimizes the total cost of a pipe system. The total cost consists of fixed (initial) and operating costs.

<table>
<thead>
<tr>
<th>Fixed (Initial) Costs</th>
<th>Operating Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Pipe cost</td>
<td>1) Pumping power costs</td>
</tr>
<tr>
<td>2) Fittings cost</td>
<td>(Electricity or fuel costs)</td>
</tr>
<tr>
<td>3) Hanger and support cost</td>
<td></td>
</tr>
<tr>
<td>4) Pump cost</td>
<td></td>
</tr>
<tr>
<td>5) Installation cost</td>
<td></td>
</tr>
</tbody>
</table>

We formulate an equation for the sum of the initial and operating costs and express the result on a **cost-per year basis**. Next differentiating the total cost with respect to diameter, we obtain the desired diameter that makes the total cost minimum.
Annual Cost

To formulate a least annual cost analysis, the initial cost of entire system must be converted into an equivalent annual cost based on a prescribed operating period. We can do this by assuming that the capital (money) is barrowed from a bank at an annual interest rate of \( i \), and that it must be repaid or amortized, in \( m \) years with yearly payments.

The annual cost to repay a loan of say $1 over \( m \) years is calculated by

\[
a = \frac{i}{1 - \left(\frac{1}{1+i}\right)^m}
\]

The parameter \( a \) is known as the amortization rate.

The initial cost \( C_i \) of a piping system (or any system) can be converted to an annual cost \( C_A \) with the following equation:

\[
C_A = a C_i = \frac{i C_i}{1 - \left[\frac{1}{1+i}\right]^m}
\]

Example: Consider a piping system that is installed for $20,000. Suppose that we fund the installation by barrowing the capital from a bank at 9% interest rate to be paid in 7 years. Calculate the annual cost.

\[
C_A = a C_i = \frac{i C_i}{1 - \left[\frac{1}{1+i}\right]^m} = \frac{0.09 \times 20000}{1 - \left[\frac{1}{1+0.09}\right]^7} = \frac{1.987 \times 20000}{1 - \left[\frac{1}{1.09}\right]^7} = $ 3974
\]

The amortization rate is 0.1987. The money paid in 7 years is 7\times3974=$27818

Initial Cost

The initial cost includes costs of pipe, pump, fittings, supports, installation, etc. To calculate optimum pipe diameter, each of all these initial costs is expressed as a function of pipe diameter for per unit pipe length.
In order to implement a least annual cost analysis, we need to fit an equation to pipe cost data. Equation fitted to the pipe cost data could be in one of the following forms:

\[ C_p = B_0 + B_1D + B_2D^2 \quad \text{or} \quad C_p = C_1D^n \quad (\$/m \quad \text{or} \quad \$/ft) \]

Where \( D \) is the pipe diameter; \( B_0, B_1, B_2, C_1 \) and \( n \) are unknown constants to be determined.

We will use second expression for pipe cost. In this expression, values of \( C_1 \) and \( n \) are determined for each pipe description. Value of \( C_1 \) varies between $22/ft^{n+1}$ and $55/ft^{n+1}$. Value of exponent \( n \) varies between 1.0 and 1.4.
Fitting Cost, $C_F$

We express the cost of fittings, valves, support, pumps and installation as a multiplier $F$ of the pipe cost as below:

$$C_F = FC_p = FC_i D^n$$

The value of the multiplier $F$ ranges from 6 to 7.

Pump cost may be included in fraction $F$ or a separate term may be included as function of pipe diameter. When pump cost is small, it is included in $F$. However, when the pipe cost is high, it is represented by a separate term.

Sum of the Pipe and Fitting Costs, $C_{PF}$

The total initial cost (pipe, fittings, supports, installation) is obtained as

$$C_{PF} = C_p + C_F = C_i D^n + FC_i D^n = (1 + F)C_i D^n \quad ($/m \text{ or } $/ft)$$

To the total cost we add an annual maintenance cost. Then the total annualized cost of pipe system is calculated as

$$C_{PT} = (a + b)(1 + F)C_i D^n \quad ($/m \text{ year})$$

a: amortization rate  
b: annual maintenance cost fraction

With this equation, all the initial cost is expressed in terms of the pipe diameter on an annual basis.
Operating Cost

The second factor in the total annual cost analysis is the cost of moving the fluid through the pipe. This cost is the cost of the energy required to pump the fluid. The energy required to pump per unit mass of fluid through the pipe line is found from the energy equation.

Write modified Bernoulli equation from inlet 1 to outlet 2 of the above piping system.

\[
\frac{P_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \left( f \frac{L V^2}{D 2g} + \sum K \frac{V^2}{2g} \right)_{\text{suction pipe}} + \left( f \frac{L V^2}{D 2g} + \sum K \frac{V^2}{2g} \right)_{\text{discharge pipe}} + \frac{g_c}{\rho g} \frac{dW}{dt}
\]

Defining the total head \( H \) as

\[
H = \frac{P g_c}{\rho g} + \frac{V^2}{2g} + z
\]

And in terms of \( H \), the above equation can be written as

\[
H_1 = H_2 + \left( f \frac{L V^2}{D 2g} + \sum K \frac{V^2}{2g} \right)_{\text{suction pipe}} + \left( f \frac{L V^2}{D 2g} + \sum K \frac{V^2}{2g} \right)_{\text{discharge pipe}} + \frac{g_c}{\rho g} \frac{dW}{dt}
\]

The preceding equation can be simplified. For this analysis, we assume that minor losses are negligible or that can be combine in some way with other friction terms. Further we assume that entire pipe line consists of only one size pipe. Rearranging and solving for pump power, we get
\[
\frac{dW}{dt} = -\dot{m} \left[ (H_2 - H_1) \frac{g}{g_c} + f \frac{L V^2}{D 2 g_c} \right]
\]

It is convenient to write the velocity in terms of mass flow rate using continuity:

\[
V = \frac{Q}{A} = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho \pi D^2}
\]

Then, above equation can be written as

\[
-\frac{dW}{dt} = \dot{m} \left[ (H_2 - H_1) \frac{g}{g_c} + \frac{8fLm^2}{\pi^2 \rho^2 D^5 g_c} \right]
\]

\(dW/dt\) is the power that must be supplied to the fluid to overcome head changes and losses. The actual motor size is \((dW/dt)/\eta\). The cost of operating the pump on a yearly basis is calculated as

\[
C_{op} = \frac{C_2 t \left( -\frac{dW}{dt} \right)}{\eta}
\]

Where

- \(dW/dt\): power supplied to the fluid (W)
- \(C_{op}\): Annual energy cost ($/year)
- \(C_2\): cost of unit energy ($/kW h)
- \(t\): time during which system operates per year (h/year)

### Total Piping Cost, \(C_T\)

Total annual cost of piping system with an amortization rate of \(a\) is obtained by summing the total pipe cost and the operating cost.

\[
C_T = LC_{pt} + C_{op} = (a + b)(1 + F)C_1 D^n L + C_2 t \left( -\frac{dW}{dt} \right)/\eta
\]

Substituting the expression for \(-dW/dt\), the total cost becomes

\[
C_T = LC_{pt} + C_{op} = (a + b)(1 + F)C_1 D^n L + \frac{\dot{m}C_2 t}{\eta} \left( H_2 - H_1 \right) \frac{g}{g_c} + \frac{8fLm^2}{\pi^2 \rho^2 D^5 g_c} \frac{C_2 t}{\eta}
\]

**Optimum economic diameter** is the diameter that minimizes the total cost. Hence, the optimum economic diameter is obtained by differentiating the total cost equation and setting the result equal to zero:
\[
\frac{\partial C_r}{\partial D} = n(a + b)(1 + F)C_1D^{(n-1)}L - 5\left(\frac{8 \bar{f} \bar{m}^3}{\pi^2 \rho^2 D^6 g_c \eta} \frac{C_2 t}{C_1 \eta \pi^2 \rho^2 g_c}\right) = 0
\]

Solving for diameter gives

\[
D^{n+5} = \frac{40 \bar{f} \bar{m}^3 C_2 t}{n(a + b)(1 + F)C_1 \eta \pi^2 \rho^2 g_c}
\]

or

\[
D_{opt} = \left[\frac{40 \bar{f} \bar{m}^3 C_2 t}{n(a + b)(1 + F)C_1 \eta \pi^2 \rho^2 g_c}\right]^{\frac{1}{n+5}}
\]

Parameter in this equation is given in the table below, which also gives some typical values.

Several features of optimum diameter equation are

1) The pipe length does not appear in the equation.
2) Viscosity of the fluid does not appear, but the density does. Viscosity influences Reynolds number, which affects the friction factor \( f \).
3) Diameter is unknown and hence a **trial and error** solution is required if the Moody diagram is used.
4) Head loss \( \Delta H \) does not appear in the equation.
5) If there were no friction effect (\( f=0 \)), an optimum diameter would not be calculated.

To avoid trial and error solution, the above equation is rearranged and \( f \) is eliminated from the right hand side of the equation. Taking the reciprocal of above equation and multiplying both sides by \( 4m/\mu \pi g_c \), we get

\[
\frac{1}{D_{opt}} = \left[\frac{n(a + b)(1 + F)C_1 \eta \pi^2 \rho^2 g_c}{40 \bar{f} \bar{m}^3 C_2 t}\right]^{\frac{1}{n+5}}
\]

Multiplying both sides by \( 4m/\mu \pi g_c \), we get
\[
\frac{4m}{\pi \mu g_c D_{opt}} = \left[ \frac{n(a + b)(1 + F)C_1 \eta \pi^2 \rho^2 g_c}{40 fm^3 C_2 t} \frac{4^{n+5} m^{n+5}}{\pi^{n+5} \mu^{n+5} g_c^{n+5}} \right]^{\frac{1}{n+5}}
\]

or
\[
\left( \frac{4m}{\pi \mu g_c D_{opt}} \right)^{n+5} = \frac{256}{10 \pi^3 g_c^4 \mu^5} \left( \frac{4m}{\pi \mu g_c} \right)^n \left( \frac{n(a + b)(1 + F)C_1 \eta \rho^2}{f C_2 t} \right)
\]

The term in the parentheses on the left hand side is recognized as the Reynolds number. Multiplying both sides by \(f\) and taking the sixth root, we get,
\[
\left( \frac{f(Re)^{n+5}}{Re} \right)^{1/6} = \left[ \frac{128}{5 \pi^3 g_c^4 \mu^5} \left( \frac{4m}{\pi \mu g_c} \right)^n \left( \frac{n(a + b)(1 + F)C_1 \eta \rho^2}{C_2 t} \right) \right]^{1/6}
\]

Also introducing roughness number, \( Ro = \frac{\varepsilon / D}{Re} = \frac{\varepsilon}{D} \left( \frac{\pi D \mu g_c}{4m} \right) = \frac{\pi \varepsilon \mu g_c}{4m} \)

For optimum economic pipe diameter, it is convenient to have a graph \( f \) vs \( (f(Re)^{n+5})^{1/6} \) with \( Ro \) as an independent parameter. This graph can be constructed for different values of \( n \). For \( n=1, 1.2 \) and \( 1.4 \) graphs are given below.
### Table 4.2: Factors in the optimum economic diameter analysis.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Dimensions (Units)</th>
<th>Typical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{opt}$</td>
<td>the optimum economic diameter</td>
<td>L (ft or m)</td>
<td>—</td>
</tr>
<tr>
<td>$m$</td>
<td>mass flow rate</td>
<td>M/T</td>
<td>—</td>
</tr>
<tr>
<td>$f$</td>
<td>friction factor</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$C_2$</td>
<td>cost of energy</td>
<td>MU/(F·L) [$/\text{(kW·hr)}$]</td>
<td>$0.04/\text{(kW·hr)}$ or $0.04/(738 \text{ ft·lbf·hr})$</td>
</tr>
<tr>
<td>$t$</td>
<td>time during which system operates per year</td>
<td>(hr/yr)</td>
<td>7880 hr/yr (10% downtime)</td>
</tr>
<tr>
<td>$n$</td>
<td>exponent of $D$ in curve fit of pipe cost data</td>
<td>—</td>
<td>1.0 to 1.4</td>
</tr>
<tr>
<td>$a$</td>
<td>amortization rate</td>
<td>1/T (1/yr)</td>
<td>1/7 to 1/20</td>
</tr>
<tr>
<td>$b$</td>
<td>yearly maintenance cost fraction</td>
<td>1/T (1/yr)</td>
<td>0.01</td>
</tr>
<tr>
<td>$F$</td>
<td>multiplier of pipe cost representing cost of fittings, pump, installation, etc.</td>
<td>—</td>
<td>6 to 7</td>
</tr>
<tr>
<td>$C_1$</td>
<td>constant in curve fit of pipe cost data</td>
<td>MU/L$^{n+1}$</td>
<td>$22/\text{ft}^{n+1}$ to $55/\text{ft}^{n+1}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>efficiency of pump</td>
<td>—</td>
<td>0.6 to 0.9</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of liquid</td>
<td>M/L$^3$ (lbm/ft$^3$ or kg/m$^3$)</td>
<td>—</td>
</tr>
</tbody>
</table>

\[
D_{opt} = \left[ \frac{40f_m^3C_2t}{n(a + b)(1 + F)C_1\eta^2\rho^2g_c} \right]^{1/(n+5)} \\
\frac{\pi \mu g_c}{4m} = \frac{\pi \mu g_c}{4m} = \left( \frac{128}{5\pi^3g_c^4} \right) \left( \frac{4m}{\pi \mu g_c} \right) ^n \left( \frac{n(a + b)(1 + F)C_1\eta^2}{C_2 t} \right) \left[ \left( \frac{f(Re)^{n+5}}{5\pi^3g_c^4} \right) \right]^{1/6}
\]
Example: A commercial steel pipeline is to be installed in a return line from a
pump to the condenser of an air conditioner in which the rejected heat is used to
preheat water to reduce energy consumption. Water is to be conveyed at a flow
rate of 3.8 lt/s. Determine the optimum economic pipe size for the installation
for given data.

\[ C_2 = \frac{0.04}{(\text{kW hr})} = \frac{0.04}{(1000 \text{ W hr})} \]
\[ C_1 = 400 \text{ m}^{2.2} \]
\[ t = 4000 \text{ hr/yr} \]
\[ F = 7.0 \]
\[ n = 1.2 \]
\[ a = 1/7 \text{ yr} \]
\[ b = 0.01 \]
\[ \eta = 75\% = 0.75 \]

For water: \( \rho = 1000 \text{ kg/m}^3 \)
\( \mu = 0.89 \times 10^{-3} \text{ Ns/m}^2 \)  
(Appendix Table B.1)
Comm. steel \( \varepsilon = 0.0046 \text{ cm} \)  
(Table 3.1)
**Example:** Select a proper pipe size for the application in the previous example. Suppose that a **schedule 40 pipe** is required.
Equivalent Length of Fittings

Minor losses may also be represented using equivalent length concept. In equivalent length concept, we replace the losses due to fittings by a pipe that cause the same loss as the fitting. The concept of equivalent length allows us to replace the minor loss factor with

\[ h_f = K \frac{V^2}{2g} \]

\[ \sum K = f \frac{Leq}{D_h} \]

**TABLE 4.3. Loss coefficient \( K \) and equivalent length-to-diameter ratio \( Leq/D \) for various fittings.**

<table>
<thead>
<tr>
<th>Fitting</th>
<th>Loss Coefficient ( K )</th>
<th>Equivalent Length-to-Diameter Ratio ( Leq/D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reentrant inlet</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Basket strainer</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>Foot valve</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>90° elbow, threaded</td>
<td></td>
<td></td>
</tr>
<tr>
<td>regular</td>
<td>1.4</td>
<td>30</td>
</tr>
<tr>
<td>long radius</td>
<td>0.75</td>
<td>20</td>
</tr>
<tr>
<td>90° elbow, flanged</td>
<td></td>
<td></td>
</tr>
<tr>
<td>regular</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>long radius</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>45° elbow, threaded, regular</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>16</td>
</tr>
<tr>
<td>45° elbow, flanged, long radius</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>Return bend, threaded, regular</td>
<td>1.5</td>
<td>50</td>
</tr>
<tr>
<td>Return bend, flanged</td>
<td></td>
<td></td>
</tr>
<tr>
<td>regular</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>long radius</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>T-joint, threaded</td>
<td></td>
<td></td>
</tr>
<tr>
<td>through flow</td>
<td>0.9</td>
<td>20</td>
</tr>
<tr>
<td>branch flow</td>
<td>1.9</td>
<td>60</td>
</tr>
<tr>
<td>T-joint, flanged</td>
<td></td>
<td></td>
</tr>
<tr>
<td>through flow</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>branch flow</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>Coupling or union</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Globe valve, fully open</td>
<td>10.0</td>
<td>340</td>
</tr>
<tr>
<td>Gate valve, fraction open:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/4</td>
<td>17.0</td>
<td>900</td>
</tr>
<tr>
<td>1/2</td>
<td>2.06</td>
<td>160</td>
</tr>
<tr>
<td>3/4</td>
<td>0.26</td>
<td>35</td>
</tr>
<tr>
<td>fully open</td>
<td>0.15</td>
<td>13</td>
</tr>
<tr>
<td>Angle valve</td>
<td>2.0</td>
<td>145</td>
</tr>
<tr>
<td>Ball valve, fully open</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Butterfly valve, fully open</td>
<td></td>
<td>40</td>
</tr>
<tr>
<td>Check valves</td>
<td></td>
<td></td>
</tr>
<tr>
<td>swing type</td>
<td>2.5</td>
<td>135</td>
</tr>
<tr>
<td>ball type</td>
<td>70.0</td>
<td></td>
</tr>
<tr>
<td>lift type</td>
<td>12.0</td>
<td></td>
</tr>
<tr>
<td>Outlet</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>
System Behavior

The methods described before based on the calculation of parameters \((Q, h_f, \text{ and } D)\) under prescribed design conditions. However, pipe systems may be operated off-design. Under such operating conditions, it is useful to know how system behaves. Hence to predict off-design behavior of a system, we graph head loss versus flow rate. Graph of head loss versus flow rate is called system curve or system behavior curve. To obtain a customary form for the system curve, we use modified Bernoulli equation.

\[
\frac{P_g c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_g c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \left( \sum f \frac{L V^2}{D 2g} + \sum K \frac{V^2}{2g} \right)
\]

Recall that total head at any section:

\[
H = \frac{P_g c}{\rho g} + \frac{V^2}{2g} + z
\]

In terms of total head \(H\), modified Bernoulli equation for a constant diameter pipe becomes:

\[
H_1 - H_2 = \left( \sum f \frac{L}{D} + \sum K \right) \frac{V^2}{2g}
\]

Substituting \(V = \frac{Q}{A} = \frac{4Q}{\pi D^2}\)

\[
\Delta H = H_1 - H_2 = \left( \sum f \frac{L}{D} + \sum K \right) \frac{16Q^2}{2\pi^2 D^4 g}
\]

or

\[
\Delta H = Q^2 \left( \frac{8(\sum fL/D + \sum K)}{\pi^2 D^4 g} \right)
\]

This is the equation of the curve \(\Delta H \text{ versus } Q\), and can be graphed for any system in which diameter is known or selected as a trial value.
Example: A piping system made of 3 nominal schedule 40 PVC pipe that conveys water from a tank. The tank level is variable and so it is desired to have information on how the flow rate will vary through the system. Generate a system curve $\Delta H$ vs $Q$ for the setup shown assuming the tank liquid level $z$ can vary from 1 to 8 ft.

Solution:
From tables;
For water $\rho=62.4$ lbm/ft$^3$ $\mu=1.9\times10^{-5}$ Ns/m$^2$ (App. Table B.1)
For 3 nominal sch 40 pipe $ID=D=0.2557$ ft $A=0.05134$ ft$^2$ (App. Table D.1)
For PVC pipe $\varepsilon=0.0$=“smooth” (Table 3.1)
TABLE 4.5. Summary of calculations for Example 4.4.

<table>
<thead>
<tr>
<th>$Q$, ft$^3$/s</th>
<th>Re</th>
<th>$f$</th>
<th>$\Delta H$, ft</th>
<th>$z$, ft</th>
<th>$Q$, gpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>5070</td>
<td>0.38</td>
<td>0.00962</td>
<td>&lt; 0</td>
<td>too low</td>
</tr>
<tr>
<td>0.1</td>
<td>50700</td>
<td>0.021</td>
<td>0.786</td>
<td>&lt; 0</td>
<td>too low</td>
</tr>
<tr>
<td>1</td>
<td>507000</td>
<td>0.013</td>
<td>70.3</td>
<td>67.4</td>
<td>too high</td>
</tr>
<tr>
<td>0.5</td>
<td>253000</td>
<td>0.0155</td>
<td>18.2</td>
<td>15.2</td>
<td>too high</td>
</tr>
<tr>
<td>0.3</td>
<td>152000</td>
<td>0.0165</td>
<td>6.66</td>
<td>3.66</td>
<td>135</td>
</tr>
<tr>
<td>0.4</td>
<td>203000</td>
<td>0.016</td>
<td>11.8</td>
<td>8.80</td>
<td>180</td>
</tr>
<tr>
<td>0.35</td>
<td>178000</td>
<td>0.016</td>
<td>9.00</td>
<td>6.00</td>
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<td>0.018</td>
<td>3.02</td>
<td>0.02</td>
<td>too low</td>
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<tr>
<td>0.23</td>
<td>117000</td>
<td>0.018</td>
<td>3.99</td>
<td>0.99</td>
<td>103</td>
</tr>
</tbody>
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FIGURE 4.11. System curve for the piping arrangement of Example 4.6.